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Math in Focus: Singapore Math National Institute

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Welcome!

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The Use of Visual Models in *Math in Focus*

Beginning Bar Models

If you can visualize, then…

- You can more easily find your way home from an unfamiliar place
- You can see positive and negative space like an interior designer
- You can 'feel' where the ball should go like a quarterback running a play
- You can find an entry point to a mathematical situation

Instruction

Leading Students to Understanding

Visualization comes out of the *process* of constructing

Construction includes forming/seeing mathematical relationships

Attributes of Instruction That Lead to Visualization

Instruction:

- Begins in the concrete experiences
- Has opportunities to connect to prior understandings
- Has opportunities to apply understanding in multiple settings
- Is given time

The concrete → pictorial → abstract approach is used to help learners to develop a strong foundation in mathematics.

Concrete Experience

Pictorial Representation

Abstract Notation

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Concrete

In the concrete stage, the **teacher** begins instruction by modeling each mathematical concept with concrete materials (e.g., two sided counters, cubes, base-ten blocks, pattern blocks, fraction bars, or geometric solids) and **students represent problems with concrete materials**.

The importance of students taking over the responsibility of creating the mathematical situations cannot be emphasized enough.

Pictorial

In this stage, the **teacher and student** transform the concrete model into a **pictorial** (semi-concrete) level, which may involve drawing pictures using circles, dots, rectangles, tallies, etc. Initially, the picture looks exactly like the concrete material then can become more abstract. The students will recognize a picture before being able to create the picture.

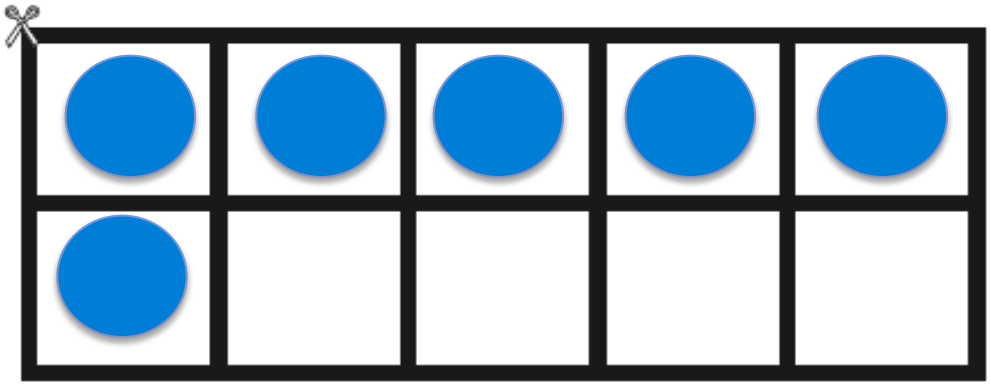
But, until the student can create the mathematical situation pictorially, true understanding will not happen.

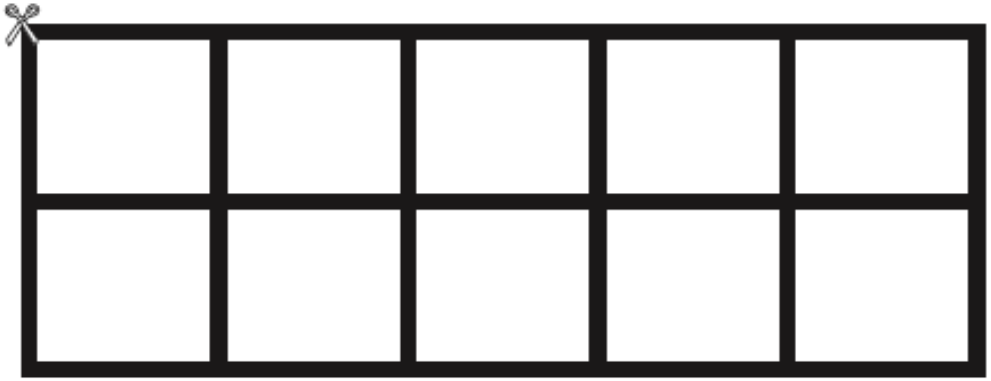
Abstract

At this stage, teachers and students record the **mathematics concept at a symbolic level,** using only numbers, notation, and mathematical symbols to represent the concept. Operation symbols (+, -, x, ÷) to indicate addition, subtraction, multiplication, or division and symbolic representations of amounts are used.

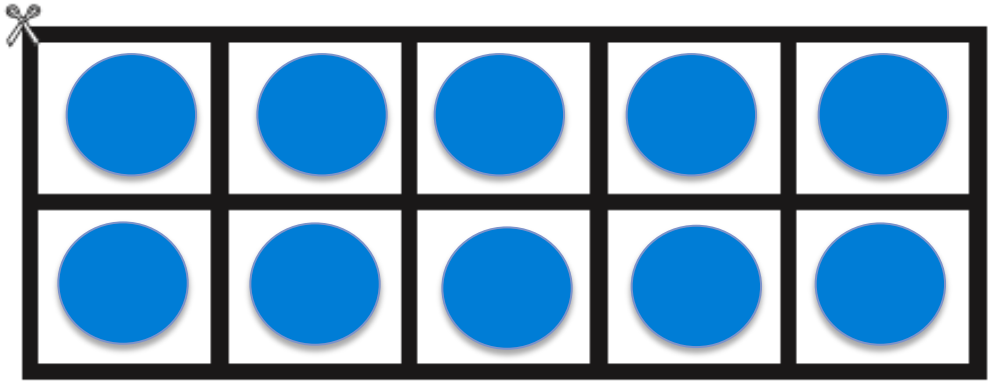
At this level, true understanding of concepts can be seen.

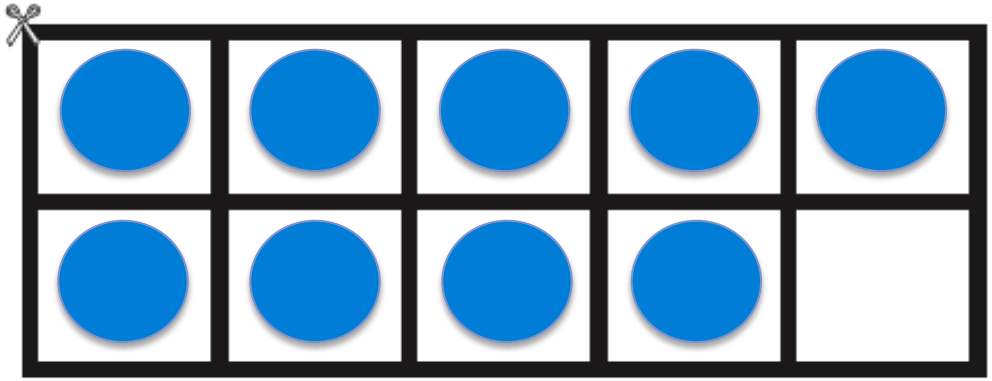
As a teacher of *Math in Focus*, you lead children to create a visual…





It’s the same amount as…





Developing the Visual: Deliberate lessons

- Each quantitative experience is planned
- Pictorial representations are purposeful
- Connections are made to more abstract representations
- Gradual Release…

Grade 1
Addition/Subtraction

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1st Grade

CHAPTER 2 Number Bonds

CHAPTER 3 Addition Facts to 10

CHAPTER 4 Subtraction Facts to 10

CHAPTER 8 Addition and Subtraction Facts to 20

CHAPTER 13 Addition and Subtraction to 40

CHAPTER 14 Mental Math Strategies

CHAPTER 17 Addition and Subtraction to 100

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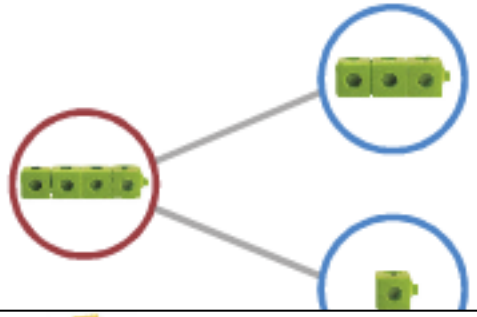
Learn

You can make number bonds with .

You can use a number train to make number bonds.

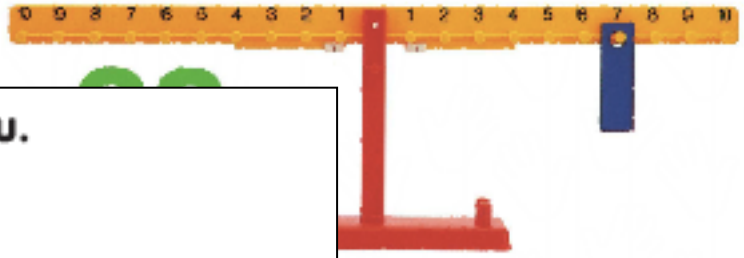
Sam put into two p

part



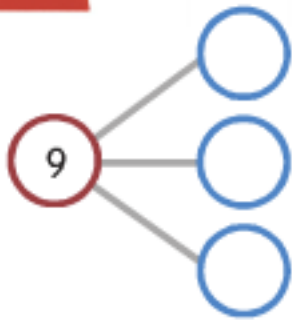
Hands-On Activity

What other numbers make 7?
Use a math balance to help you.



Use or a math balance to help you.

1 Find three numbers that make 9.

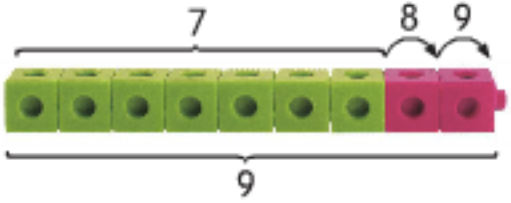


Learn

You can count on to find how many more.

What is 2 **more than** 7?

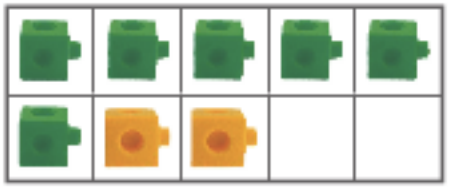
More than means add



9 is 2 more than 7.

7, 8,

What is 2 more than 6?



6, ,

is 2 more than 6.

2 a
7 is

Learn

Number bonds can help you add.

How many toy cars are there in all?

$3 + 2 = 5$

$2 + 3 = 5$

There are 5 toy cars in all.

You can add in any order.

$3 + 2 = 2 + 3$

Hands-On Activity

Use ● and ▦.

1. 9 – 3 = ?
   Put 9 ● on the ▦.
   Then take away 3 ●.
   9 – 3 = ☐

Solve.

2. 10 – 5 = ☐

3. 8 – 7 =

WORKING TOGETHER Game

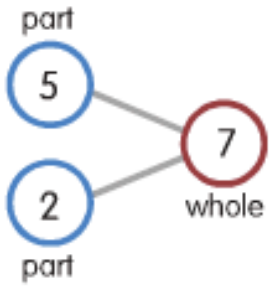
What's Hidden?

Players: 3-4
You need:

How to play:

Learn

Addition and subtraction are related.



How many balls of wool are yellow?

7 – 2 = 5

How many balls of wool are blue?

7 – 5 = 2

How many balls of wool are there in all?

2 + 5 = 7 or 5 + 2 = 7

7 – 2 = 5 7 – 5 = 2 2 + 5 = 7 5 + 2 = 7

This is a **fact family**.

Each fact in a fact family has the same parts and whole.

Learn

Read and understand a word pr

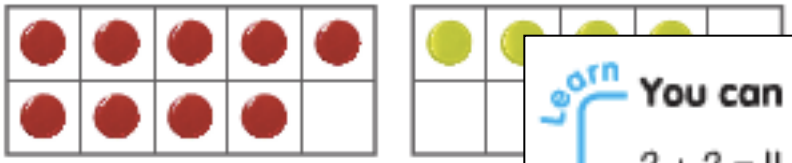
Nora and Keis
Nora has 7 or
How many or

9 – 7 = 2

Keisha has 2

Make a 10.
Then add.

1. $9 + 4 =$ ☐



2. $7 + 9 =$ ☐

3. $9 + 8 =$ ☐

4.

Learn

You can use doubles plus one facts to add.

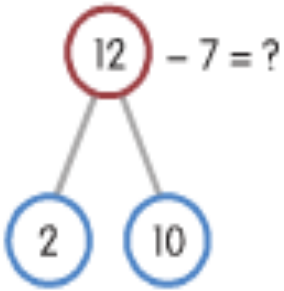
$2 + 2 = 4$ is a doubles fact.

Learn

You can subtract by grouping into a 10 and ones.

Shawn makes 12 stars.
He gives 7 to Gina.
How many stars does Shawn have left?

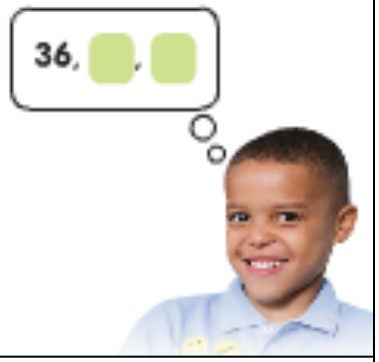
Step 1 Group 12 into a 10 and ones.
$12 = 10 + 2$



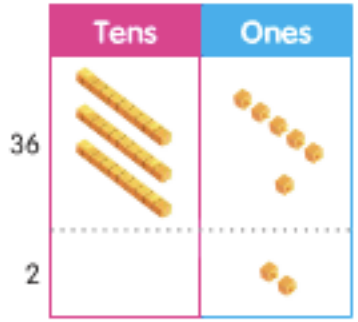
If a child has a pile of 10 counters and a pile of 2 counters
and is asked to pull out 7,
from which pile will he choose?

36 + 2 = ?

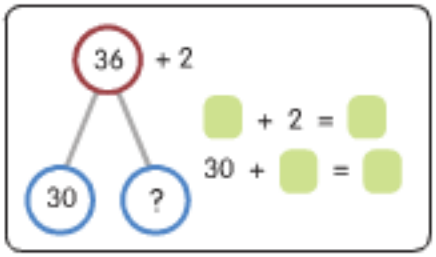
Method 1 Count on from the greater number.



Method 2 Use a place-value chart.



	Tens	Ones
	3	6
+		2



So, 36 + 2 = .

Addition w/o regrouping
Addition w/ regrouping
Subtraction w/o regrouping
Subtraction w/o regrouping

WORKING TOGETHER Game

Race to 40!

Players: 4–6
You need:
- red beans
- green beans
- a number cube
- a place-value chart for each player

How to play:

STEP 1 The red beans stand for tens and the green beans stand for ones.

STEP 2 You roll the num cube to get a nu

STEP 3 Put this number of green beans on your place-value chart.

STEP 4 The other players take turns to repeat STEP 2 and STEP 3.

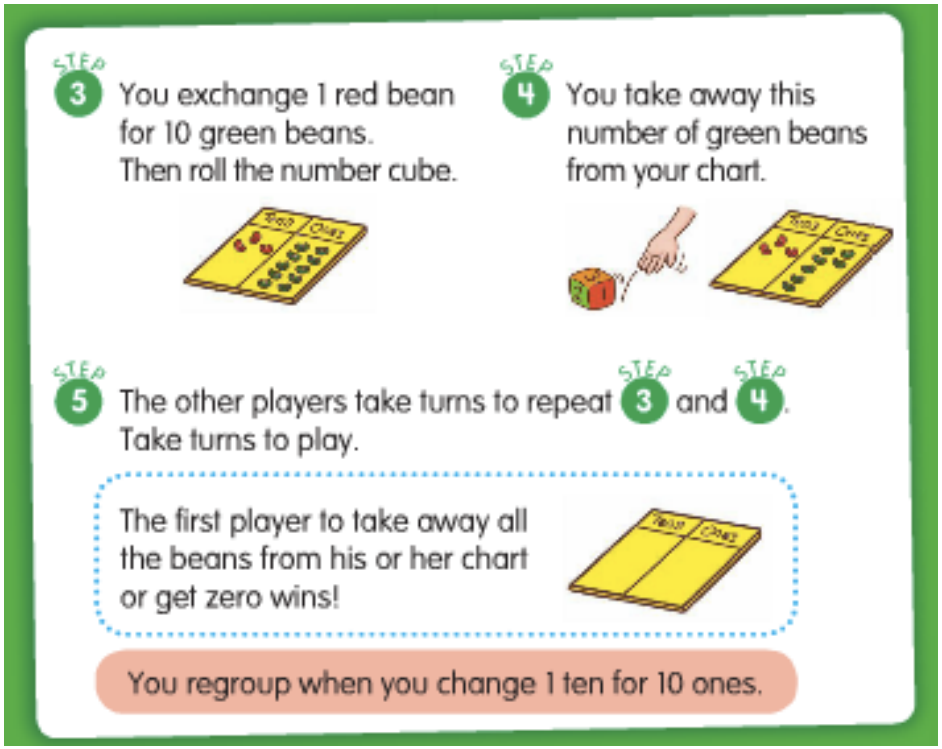
STEP 5 When it is your turn again, roll the number cube. Add to the number of beans on your chart.

STEP 6 If you get 10 or more green beans, exchange 10 of them for 1 red bean.

You **regroup** when you change 10 ones for 1 ten.

The first player to get 4 red beans or 4 tens wins!

How would this help to develop place value understanding?



How could one help students make connections moving forward?

Learn

You can add o

55 + 4 = ?

Method 1 Cou
the

5

Method 2 Use

Tens

55

4

So, 55 + 4 = 59

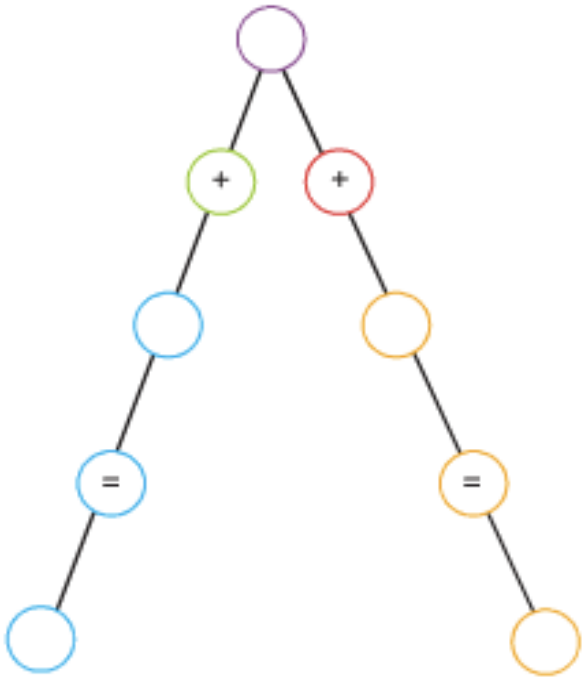
CRITICAL THINKING SKILLS

Put On Your Thinking Cap!

PROBLEM SOLVING

Use each number once.

14 25 49 39 74



ON YOUR OWN

Go to Workbook B:
Put on Your Thinking Cap!
pages 173–174

in different ways.

r number.

, 46, 45

tract the ones.

nes

8
3
5

8 ones – 3 ones
= 5 ones

tract the tens.

nes

8
3
5

4 tens – 0 tens
= 4 tens

- The lesson is not about the teacher hitting the exact parts of the lesson…

- It's about kids walking away challenged and thinking about things. That's the reason there are multiple days to build concrete to pictorial to abstract…because the kids have to make sense of something they didn't know before.

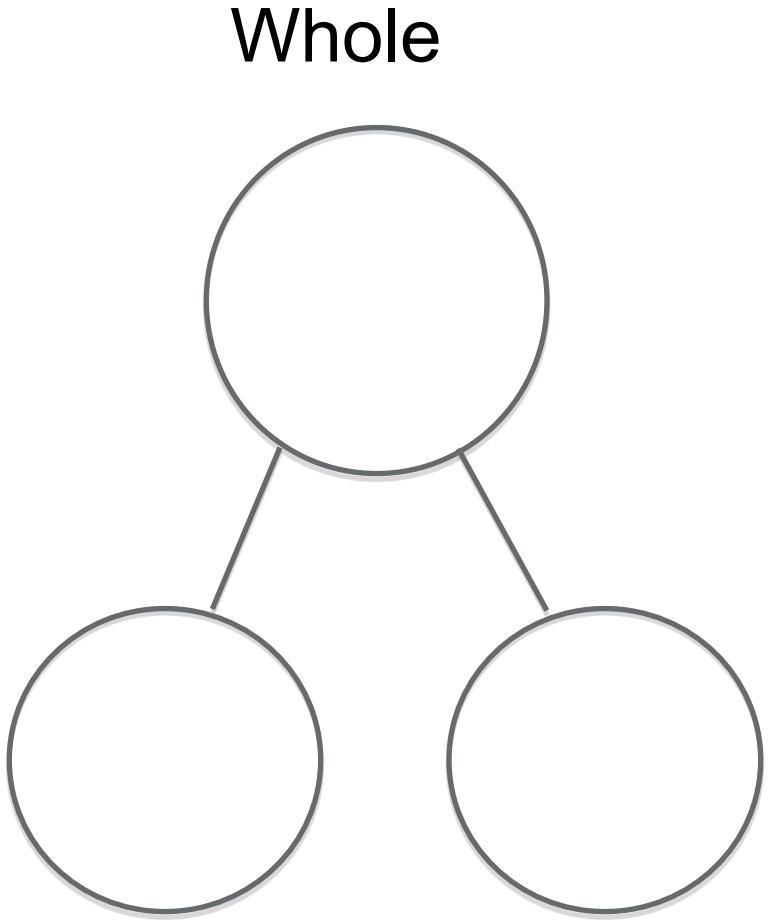
Visualization at Work

Opportunities in *Math in Focus* to Build Understanding

Number Bonds

Number Bonds

- part/whole
- decomposing numbers
- conservation of number
- relationship between addition & subtraction
- commutative property



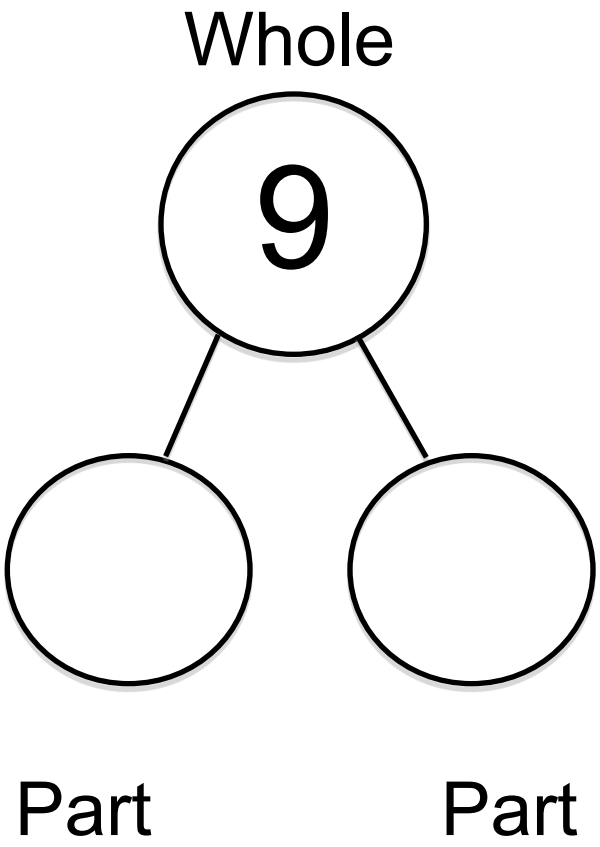
Combos of 8
Different from neighbor

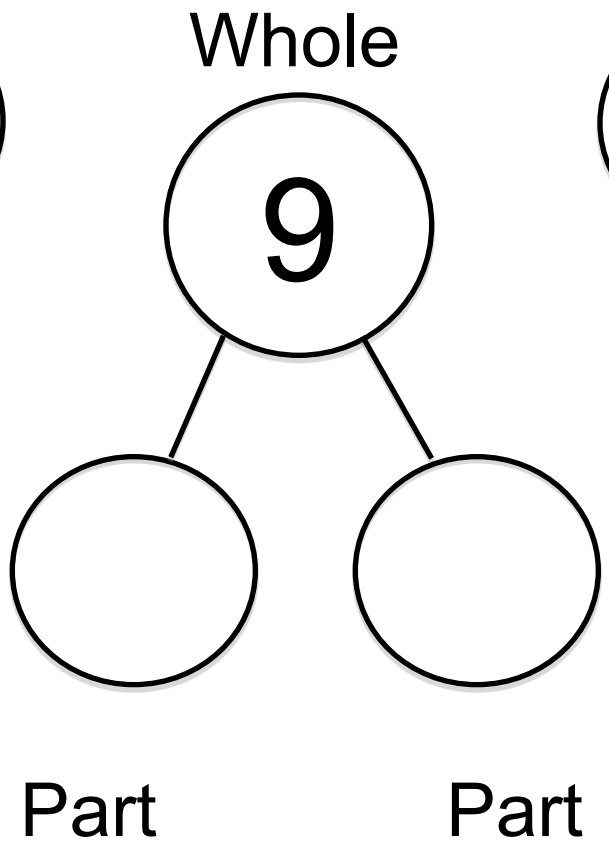
Number Bonds Task

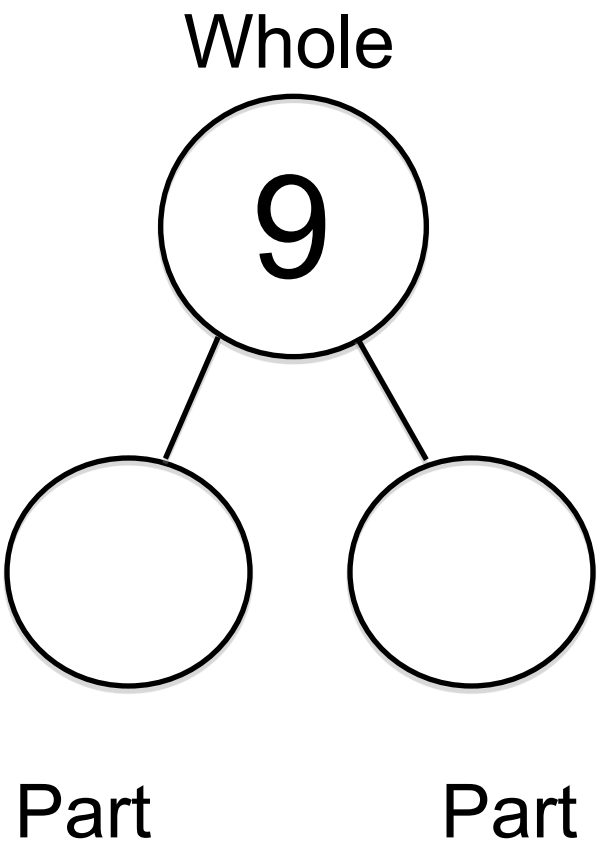
Use your age, add the digits
If your number is a 1-digit number, use it
If your number is a 2-digit number, use the ones place

Use this number as the “whole”

Create a number bond for that number in at least 3 different ways

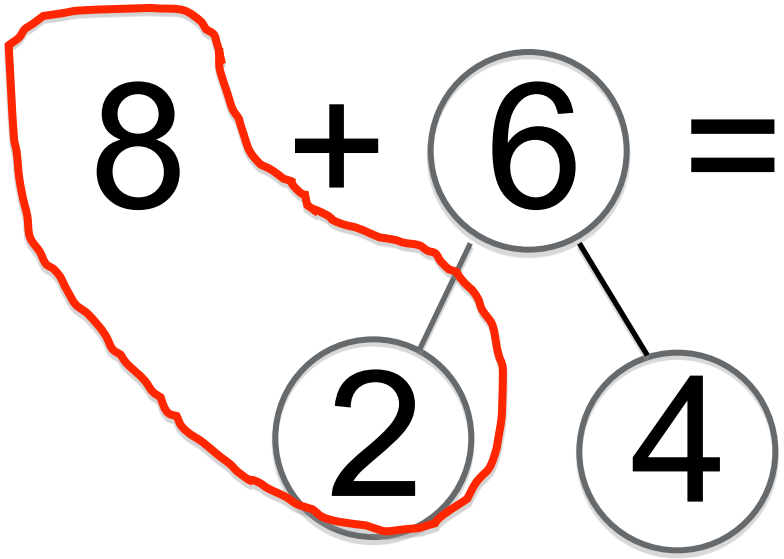






Number Bonds Extend to Mental Math

This strategy in *Math In Focus* is “You can add by Making a 10”



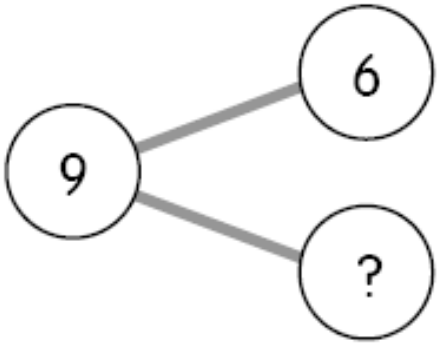
8 + 2 = 10

10 + 4 = 14

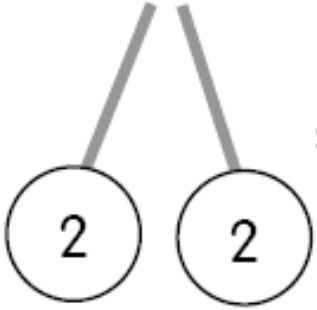
So, 8 + 6 = 14

Number Bonds

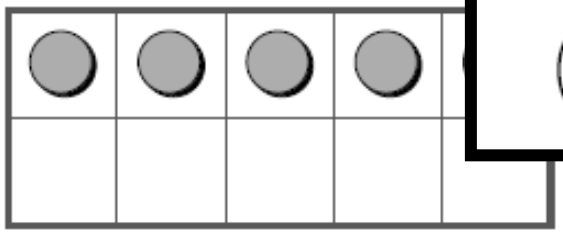
6 and _____ make 9.

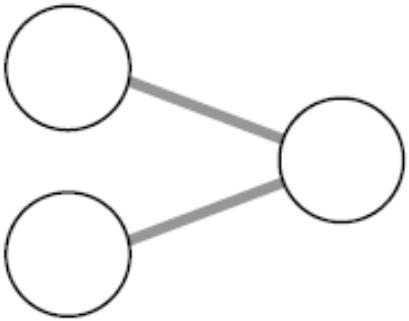


4 + 8 = 2 + 10



= _____





5 + 1 = _____

Number Bonds

Grade 3, Chpt 16

Learn

Subtract time with regrouping.

Kyle bikes for 4 hours 30 minutes.
Joey bikes for 2 hours 50 minutes.
How much longer does Kyle bike than Joey?

4 h 30 min − 2 h 50 min = ?

STEP 1 Regroup 4 h 30 min.

4 h 30 min = 3 h 90 min

You cannot subtract 50 minutes from 30 minutes. So regroup 4 hours 30 minutes.

STEP 2 Subtract.

3 h 90 min − 2 h 50 min = 1 h 40 min

Kyle bikes 1 hour 40 minutes longer than Joey.

Place Value

Place Value is a common visual, that when developed correctly, deters kids from doing this:

$$\begin{array}{r} 300 \\ -\ 299 \\ \hline \end{array} \qquad \begin{array}{r} 56 \\ -\ 39 \\ \hline \end{array} \qquad \begin{array}{r} 12 \\ -\ 7 \\ \hline \end{array}$$

Place Value Charts

Learn

You can use models to show numbers to 20.

13 = 1 ten 3 ones

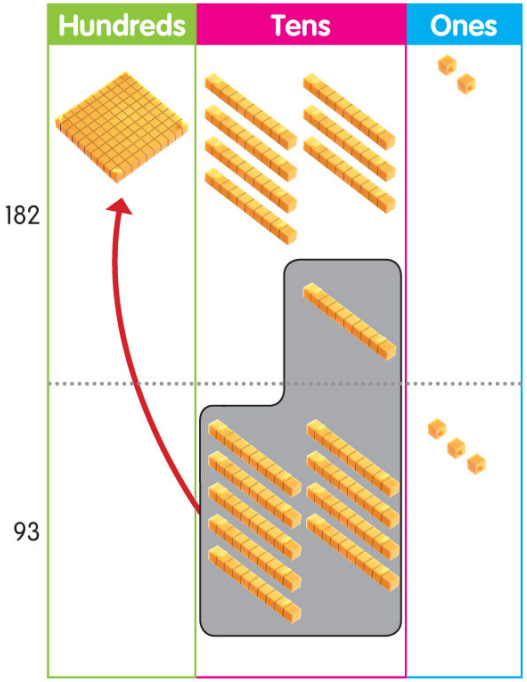
Tens	Ones
1	3
	

Place Value Charts

$$\begin{array}{r} 182 \\ +\ 93 \\ \hline \end{array}$$

Learn **You can add using base-ten blocks and a place-value chart to regroup tens.**

182 + 93 = ?



Step 1
Add the ones.

$$\begin{array}{r} 1\ 8\ 2 \\ +\ \ 9\ 3 \\ \hline \ \ \ \ 5 \end{array}$$

2 ones + 3 ones = 5 ones

Step 2
Add the tens.

$$\begin{array}{r} {\scriptstyle 1\ \ \ } \\ 1\ 8\ 2 \\ +\ \ 9\ 3 \\ \hline 7\ 5 \end{array}$$

8 tens + 9 tens = 17 tens

Regroup the tens.

17 tens = 1 hundred 7 tens

Solve the following with a partner:

- One person will use Base 10 Block as intended

- On person will use Base Ten Units in a way that mimics our number system

Place Value Task:

Draw the number 431 on your place value chart

Write the digits above the correct place

Rearrange the digits to create the smallest number possible, draw that number

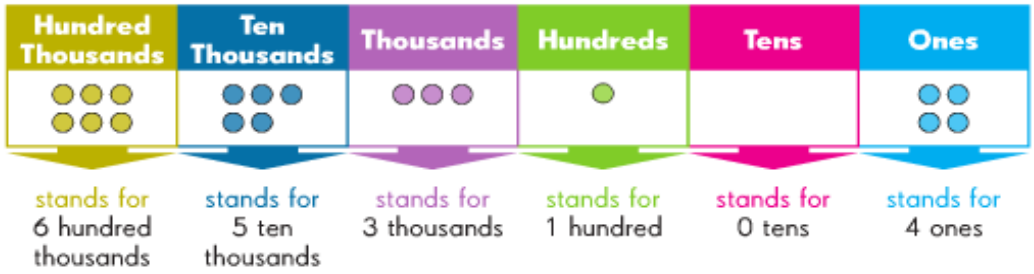
Regroup your materials/drawings so that your group is different from your neighbor's but retains the same value

Share your answer

Learn

Write numbers in standard form and word form.

What is the number in standard form and word form?

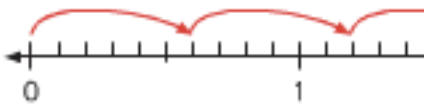


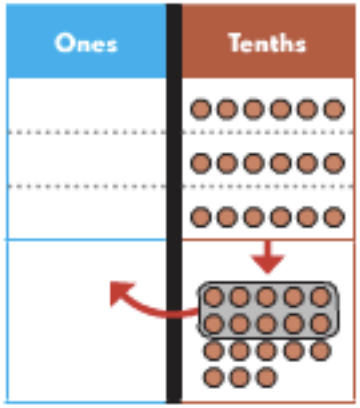
	Standard Form	Word Form
6 hundred thousands	600,000	six hundred thousand
5 ten thousands	50,000	fifty thousand
3 thousands	3,000	three thousand
1 hundred	100	one hundred
0 tens	0	
4 ones	4	four

Number in standard form: 653,104

Number in word form: six hundred fifty-three thousand, one hundred four

2





Learn

Multiply decimals by 10.

Look at these examples.

	Hundreds	Tens	Ones	Tenths	Hundredths
12		●	●●		
12 × 10					
1					
1 × 10					
0.1					
0.1 × 10					
0.12					
0.12 × 10					

	Hundreds	Tens	Ones	Tenths	Hundredths
12		1	2		
12 × 10					
1					
1 × 10					
0.1					
0.1 × 10					
0.12					
0.12 × 10					

What happens to the digits of the decimal when it is multiplied by 10?
Each digit moves 1 place to the left in the place-value chart.

Geometry

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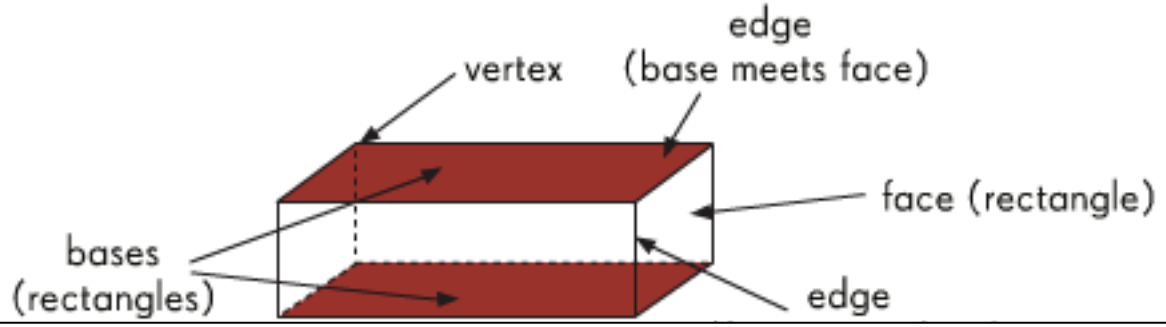
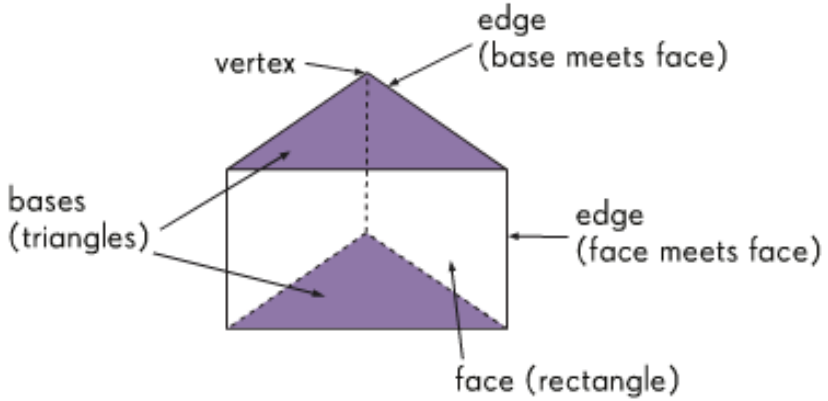
The bases of this prism are rectangles. This prism is called a **rectangular prism**.



Any pair

A r

The bases of this prism are triangles. This prism is called a **triangular prism**.

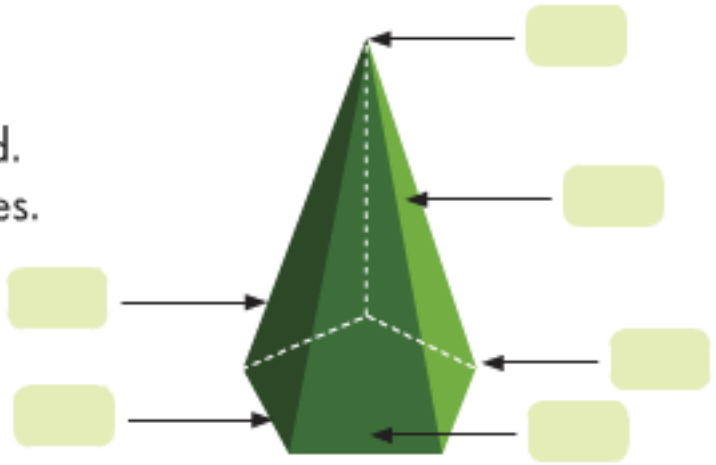


Can you find other pairs of bases in this triangular prism? Explain your answer.

A triangular prism has 5 faces, 9 edges, and 6 vertices.

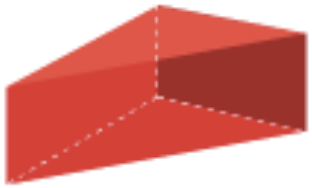
Complete.

4 Name the parts of the pyramid.
Identify the shapes of the faces.



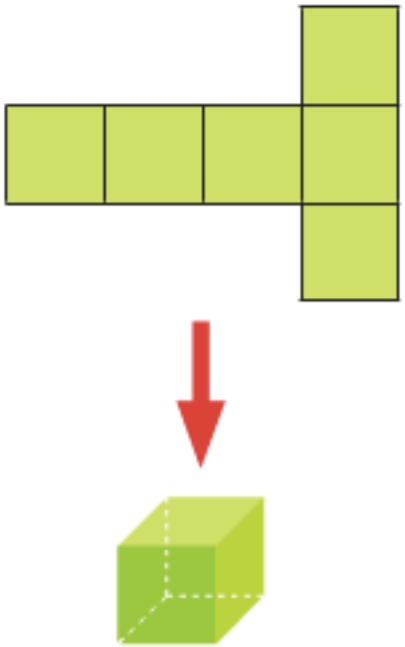
Explain.

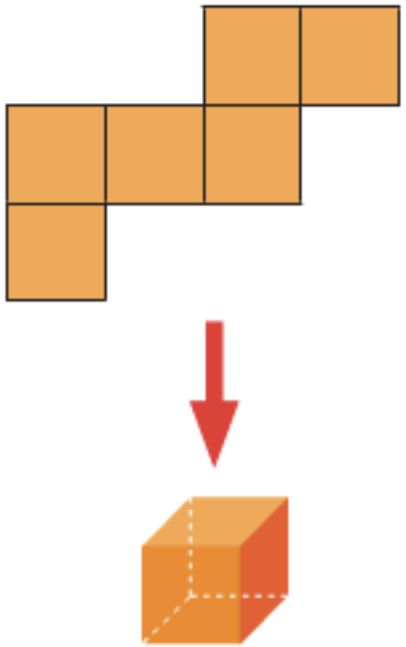
5 Explain why the figure is not a pyramid.



A **net** is a plane figure that can be folded to make a solid figure. More than one net may form the same solid figure.

These are two more nets of a cube.





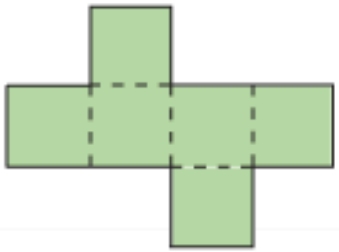
Hands-On Activity

IDENTIFYING A CUBE FROM A NET

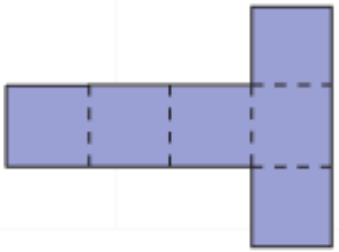
Work in pairs.

STEP 1 Trace and cut out each figure.

a)



b)



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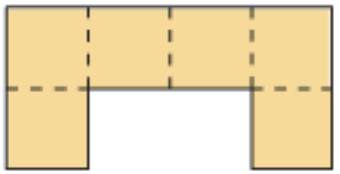
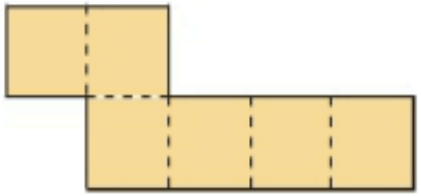


STEP 2 Try and fold them into cubes.

Math Journal Can you fold all the above figures into cubes? Discuss with your partner and explain your thinking.

Decide if each net will form a cube. Answer Yes or No.

9



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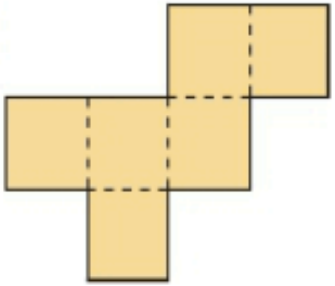

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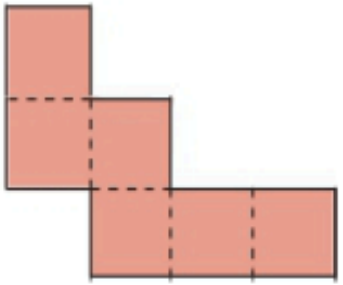

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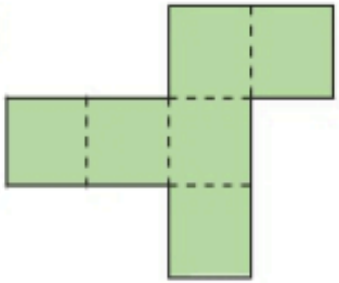

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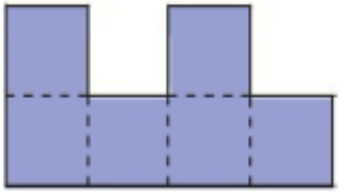

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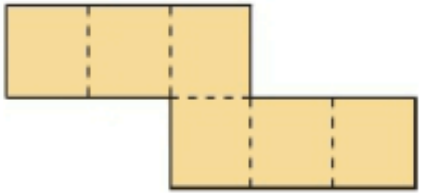


Visualization

Finding an Entry Point to Problem Solving

Bar Modeling

Miss Lucy has 27 students in her morning ballet class. She has 39 students in her afternoon ballet class.

How many students does she have in both classes?

Try it!

____ men and ___ women go to a concert.

How many adults are at the concert?

Luke has 83 toy cars. His brother gives him 52 more toy cars.

How many toy cars does he have altogether?

Try it!

Daniel has ____ craft sticks for his project. He needs ____ more craft sticks.

How many craft sticks does he need for his project?

102 children at a swimming pool do not wear goggles. 23 more children wear goggles than those who do not wear goggles.

How many children wear goggles?

Try it!

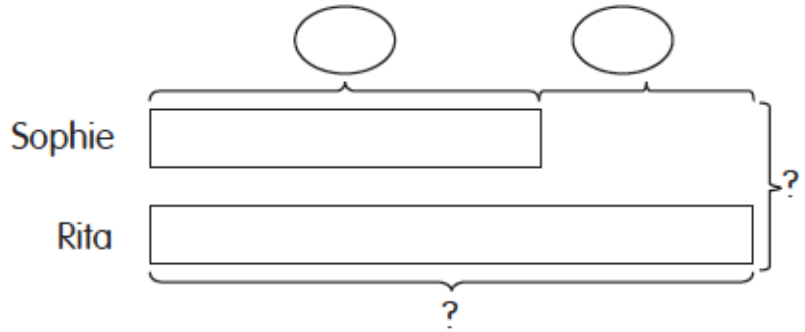
Alice made ____ ham sandwiches for a party. She made ____ fewer cheese sandwiches than ham sandwiches for the party.

How many cheese sandwiches did Alice make?

Sophie has 356 stamps in her collection. Rita has 192 stamps more than Sophie.

a. How many stamps does Rita have?

b. How many stamps do they have in all?



Try it!

Mr. Kim has 78 boxes of apples and 130 boxes of oranges. He sells some boxes of oranges.

Now he has 159 boxes of apples and oranges left.

a. How many boxes of apples and oranges did Mr. Kim have at first?

b. How many boxes of oranges did Mr. Kim sell?

Using Bar Models: Addition and Subtraction

Solve using addition and subtraction.

1

How many marbles are there?

$42 + 21 = 63$

There are 63 marbles.

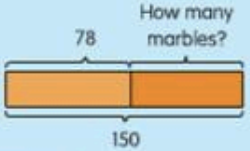
Check the answer.

$63 - 21 = 42$

$63 - 42 = 21$

The answer is correct.

2



$150 - 78 = 72$

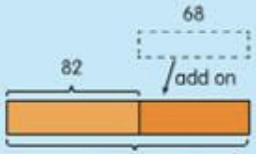
There are 72 marbles.

Check the answer.

$72 + 78 = 150$

The answer is correct.

3



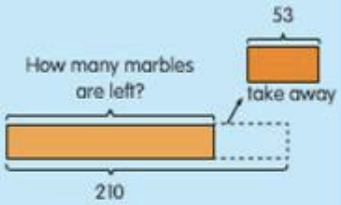
How many marbles are there in all?

$82 + 68 = 150$

There are 150 marbles in all.

Check the answer.

4



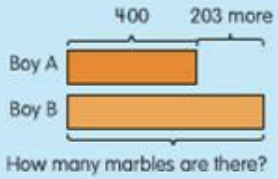
$210 - 53 = 157$

There are 157 marbles left.

Check the answer.

Draw bar models for one-step addition and subtraction problems.

5



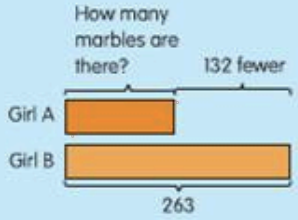
How many marbles are there?

$400 + 203 = 603$

Boy B has 603 marbles.

Check the answer.

6



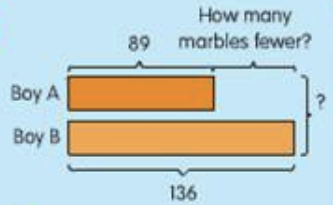
$263 - 132 = 131$

Girl A has 131 marbles.

Check the answer.

Draw bar models for two-step addition and subtraction problems.

7



a $136 - 89 = 47$

Boy A has 47 fewer marbles than Boy B.

b $89 + 136 = 225$

They have 225 marbles in all.

Check the answer.

ON YOUR OWN

Go to Workbook A: Chapter Review/Test, pages 95–98

Farah has 8 vases.
She puts 5 flowers in each vase.

How many flowers does Farah have in all?

Ben has 35 leather strips. He uses 5 strips for each necklace he makes.

How many necklaces does he make?

Try it!

Lily sews 24 dresses for her dolls.
Each doll gets 3 dresses.

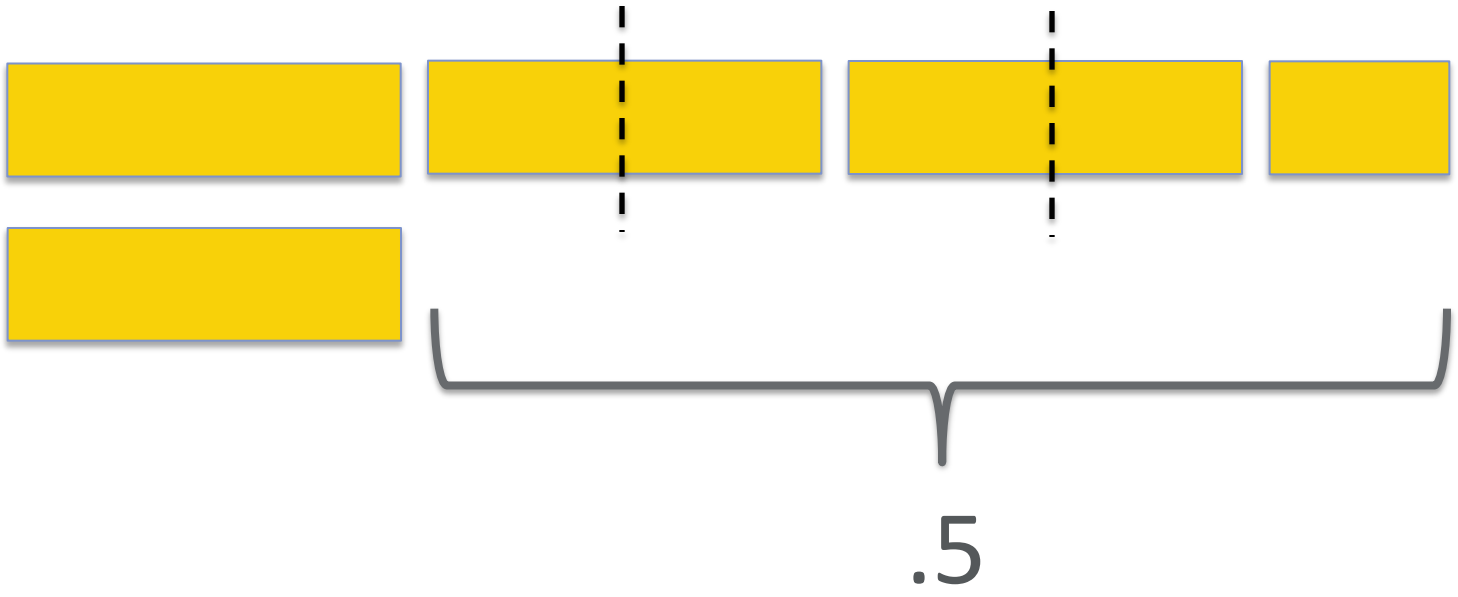
How many dolls does Lily have?

Ricardo spends 8/9 of an hour reading the newspaper. He spends 1/4 of the time reading the world news and splits the remaining time equally between the sports and the comics.

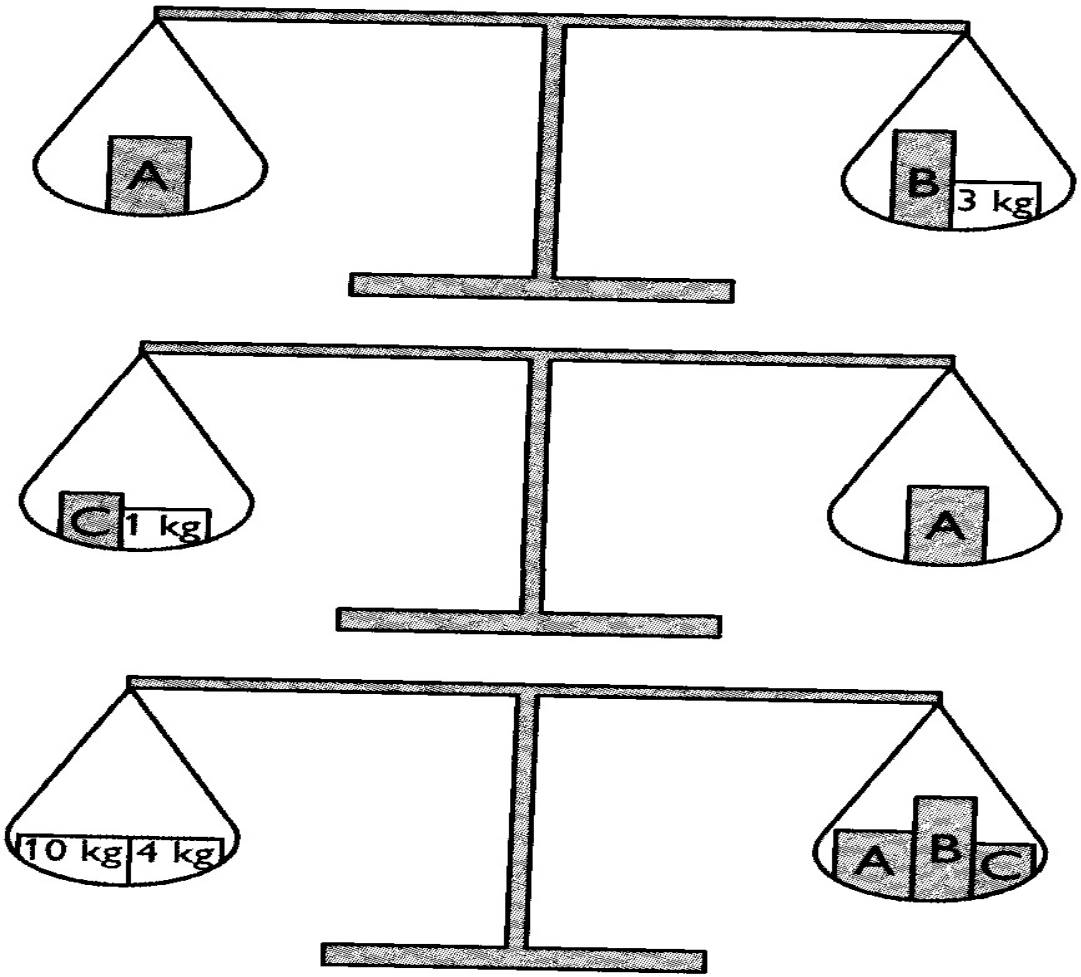
How much time does he spend reading the comics?

3.5 times a positive number is equal to the sum of the positive number and 0.5.

What is the positive number?



Find the weight of A.



The cost of 3 pairs of socks and 4 T-shirts is $132. Each T-shirt costs twice as much as a pair of socks.

What is the cost of each pair of socks?
What is the cost of each T-shirt?

Only when students can create their own visual and replicate it, will you really know whether they understand a concept.